MATHEMATICS

Part-A

Q.1 to Q. 20 are Single Correct Type Questions (+4, -1)

All Questions are Compulsory

1. $\lim\limits_{x\to\frac{\pi}{2}}\left(\dfrac{\int_{x^3}^{(\pi/2)^3}\left(\sin\left(2t^{1/3}\right)+\cos\left(t^{1/3}\right)\right)dt}{\left(x-\frac{\pi}{2}\right)^2}\right)$ is equal to

(1) $\dfrac{9\pi^2}{8}$ (2) $\dfrac{3\pi^2}{2}$ (3) $\dfrac{5\pi^2}{9}$ (4) $\dfrac{11\pi^2}{10}$

[JEE Main, 9th April 2024, Evening Shift]

FPR: 4

Class: XII

Chapter: Definite Integration

Subtopic: Leibnitz Theorem

2. The sum of the coefficient of $x^{2/3}$ and $x^{-2/5}$ in the binomial expansion of $\left(x^{2/3}+\dfrac{1}{2}x^{-2/5}\right)^9$ is

(1) 63/16 (2) 21/4 (3) 69/16 (4) 19/4

[JEE Main, 9th April 2024, Evening Shift]

FPR: 3

Class: XI

Chapter: Binomial Theorem

Subtopic: General Term and Coefficients

MATHEMATICS

3. The area (in square units) of the region enclosed by the ellipse $x^2+3y^2=18$ in the first quadrant below the line $y=x$ is

(1) $\sqrt{3}\pi+\frac{3}{4}$ (2) $\sqrt{3}\pi+1$ (3) $\sqrt{3}\pi$ (4) $\sqrt{3}\pi-\frac{3}{4}$

[JEE Main, 9th April 2024, Evening Shift]

FPR: 4

Class: XI

Chapter: Ellipse

Subtopic: Auxiliary Circle and Parametric Equation

4. Let $\alpha,\beta;\alpha>\beta$, be the roots of the equation $x^2-\sqrt{2}x-\sqrt{3}=0$. Let $P_n=\alpha^n-\beta^n, n\in\mathbb{N}$. Then $\left(11\sqrt{3}-10\sqrt{2}\right)P_{10}+\left(11\sqrt{2}+10\right)P_{11}-11P_{12}$ is equal to

(1) $10\sqrt{3}P_9$ (2) $11\sqrt{2}P_9$ (3) $10\sqrt{2}P_9$ (4) $11\sqrt{3}P_9$

[JEE Main, 9th April 2024, Evening Shift]

FPR: 2

Class: XI

Chapter: Quadratic Equation

Subtopic: Newton's Formula

MATHEMATICS

5. Let $\vec{a}=2\hat{i}+\alpha\hat{j}+\hat{k},\vec{b}=-\hat{i}+\hat{k},\vec{c}=\beta\hat{j}-\hat{k}$, where α and β are integers and $\alpha\beta=-6$. Let the values of the ordered pair (α,β) for which the area of the parallelogram of diagonals $\vec{a}+\vec{b}$ and $\vec{b}+\vec{c}$ is $\frac{\sqrt{21}}{2}$, be (α_1,β_1) and $(\alpha_2,\ \beta_2\ )$. Then $\alpha_1^2+\beta_1^2-\alpha_2\beta_2$ is equal to:

(1) 19 (2) 17 (3) 24 (4) 21

[JEE Main, 9th April 2024, Evening Shift]

FPR: 4

Class: XII

Chapter: Vector Algebra

Subtopic: Cross Product

6. Between the following two statements :

Statement I : Let $\vec{a}=\hat{i}+2\hat{j}-3\hat{k}$ and $\vec{b}=2\hat{i}+\hat{j}-\hat{k}$. Then the vector $\vec{r}$ satisfying $\vec{a}\times\vec{r}=\vec{a}\times\vec{b}$ and $\vec{a}\cdot\vec{r}=0$ is of magnitude $\sqrt{10}$.

Statement II : In a triangle $ABC,\cos2A+\cos2B+\cos2C\geq-\frac{3}{2}$.

(1) Statement I is correct but Statement II is incorrect.

(2) Statement I is incorrect but Statement II is correct.

(3) Both Statement I and Statement II are correct.

(4) Both Statement I and Statement II are incorrect.

[JEE Main, 9th April 2024, Evening Shift]

FPR: 5

Class: XII

Chapter: Vector Algebra

Subtopic: Miscellaneous/Mixed

MATHEMATICS

7. Let z be a complex number such that the real part of $\frac{z-2i}{z+2i}$ is zero. Then, the maximum value of $|z-(6+8i)|$ is equal to

(1) 10 (2) 12 (3) 8 (4) ∞

[JEE Main, 9th April 2024, Evening Shift]

FPR: 4

Class: XI

Chapter: Complex Numbers

Subtopic: Triangular Inequality

8. $\lim\limits_{x\to 0}\frac{e-(1+2x)^{\frac{1}{2x}}}{x}$ is equal to

(1) 0 (2) e (3) $e-e^2$ (4) $\frac{-2}{e}$

[JEE Main, 9th April 2024, Evening Shift]

FPR: 2

Class: XII

Chapter: Limits

Subtopic: Series Expansion

MATHEMATICS

9. Let the foci of a hyperbola H coincide with the foci of the ellipse $E:\frac{(x-1)^2}{100}+\frac{(y-1)^2}{75}=1$ and the eccentricity of the hyperbola H be the reciprocal of the eccentricity of the ellipse E. If the length of the transverse axis of H is α and the length of its conjugate axis is β, then $3\alpha^2+2\beta^2$ is equal to

(1) 205 (2) 225 (3) 242 (4) 237

[JEE Main, 9th April 2024, Evening Shift]

FPR: 2

Class: XI

Chapter: Hyperbola

Subtopic: General Terms

10. The integral $\int_{1/4}^{3/4}\cos\left(2\cot^{-1}\sqrt{\frac{1-x}{1+x}}\right)dx$ is equal to

(1) 1/2 (2) –1/2 (3) –1/4 (4) ¼

[JEE Main, 9th April 2024, Evening Shift]

FPR: 4

Class: XII

Chapter: Definite Integration

Subtopic: Direct Integration

MATHEMATICS

11. If $\log_e y = 3\sin^{-1}x$, then $\left(1-x^2\right)y'' - xy'$ at $x = \frac{1}{2}$ is equal to

(1) $3e^{\pi/6}$ (2) $3e^{\pi/2}$ (3) $9e^{\pi/6}$ (4) $9e^{\pi/2}$

[JEE Main, 9th April 2024, Evening Shift]

FPR: 2

Class: XII

Chapter: Methods of Differentiation

Subtopic: Differentiation Based on Basic Rules(Product, Quotient and Chain)

12. If the variance of the frequency distribution

x	c	2 c	3 c	4 c	5 c	6 c
f	2	1	1	1	1	1

is 160, then the value of $c \in N$ is

(1) 6 (2) 7 (3) 5 (4) 8

[JEE Main, 9th April 2024, Evening Shift]

FPR: 1

Class: XI

Chapter: Statistics

Subtopic: Mean and Variance

MATHEMATICS

13. Let the range of the function $f(x)=\frac{1}{2+\sin 3x+\cos 3x}, x\in\mathbb{R}$ be $[a,b]$. If α and β are respectively the A.M. and the G.M. of a and b, then $\frac{\alpha}{\beta}$ is equal to

(1) $\sqrt{2}$ (2) π (3) $\sqrt{\pi}$ (4) 2

[JEE Main, 9th April 2024, Evening Shift]

FPR: 1

Class: XI

Chapter: Trigonometric Ratios and Identities

Subtopic: Trigonometric Functions

14. Let $B=\begin{bmatrix}1 & 3\\ 1 & 5\end{bmatrix}$ and A be a 2×2 matrix such that $AB^{-1}=A^{-1}$. If $BCB^{-1}=A$ and $C^4+\alpha C^2+\beta I=O$, then $2\beta-\alpha$ is equal to

(1) 16 (2) 8 (3) 2 (4) 10

[JEE Main, 9th April 2024, Evening Shift]

FPR: 5

Class: XII

Chapter: Matrices

Subtopic: Characteristic Equation of a Square Matrix

MATHEMATICS

15. The value of the integral $\int_{-1}^{2} \log_e\left(x+\sqrt{x^2+1}\right)dx$ is

(1) $\sqrt{2}-\sqrt{5}+\log_e\left(\frac{7+4\sqrt{5}}{1+\sqrt{2}}\right)$

(2) $\sqrt{5}-\sqrt{2}+\log_e\left(\frac{7+4\sqrt{5}}{1+\sqrt{2}}\right)$

(3) $\sqrt{2}-\sqrt{5}+\log_e\left(\frac{9+4\sqrt{5}}{1+\sqrt{2}}\right)$

(4) $\sqrt{5}-\sqrt{2}+\log_e\left(\frac{9+4\sqrt{5}}{1+\sqrt{2}}\right)$

[JEE Main, 9th April 2024, Evening Shift]

FPR: 5

Class: XII

Chapter: Definite Integration

Subtopic: Definite Integration by Parts

16. If an unbiased dice is rolled thrice, then the probability of getting a greater number in the i^{th} roll than the number obtained in the $(i-1)^{th}$ roll, i = 2,3, is equal to

(1) 1/54 (2) 5/54 (3) 2/54 (4) 3/54

[JEE Main, 9th April 2024, Evening Shift]

FPR: 1

Class: XII

Chapter: Probability

Subtopic: Classical Definition

MATHEMATICS

17. Let $a, ar, ar^2, \ldots\ldots$ be an infinite G.P. If $\sum_{n=0}^{\infty} ar^n = 57$ and $\sum_{n=0}^{\infty} a^3 r^{3n} = 9747$, then $a + 18r$ is equal to

(1) 46 (2) 38 (3) 27 (4) 31

[JEE Main, 9th April 2024, Evening Shift]

FPR: 4

Class: XI

Chapter: Sequence and Series

Subtopic: Geometric Progression

18. Two vertices of a triangle ABC are A(3, –1) and B(–2, 3), and its orthocentre is P(1, 1). If the coordinates of the point C are (α, β) and the centre of the circle circumscribing the triangle PAB is (h, k), then the value of $(\alpha + \beta) + 2(h + k)$ equals

(1) 5 (2) 51 (3) 15 (4) 81

[JEE Main, 9th April 2024, Evening Shift]

FPR: 3

Class: XI

Chapter: Straight Line

Subtopic: Centres of a Triangle

MATHEMATICS

19. Let $\int_0^x \sqrt{1-\left(y'(t)\right)^2}\,dt = \int_0^x y(t)\,dt, 0 \le x \le 3, y \ge 0, y(0) = 0$. Then at $x = 2, y'' + y + 1$ is equal to

(1) 1 (2) 2 (3) $\sqrt{2}$ (4) ½

[JEE Main, 9th April 2024, Evening Shift]

FPR: 2

Class: XII

Chapter: Definite Integration

Subtopic: Leibnitz Theorem

20. Consider the line L passing through the points $(1,2,3)$ and $(2,3,5)$. The distance of the point $\left(\frac{11}{3}, \frac{11}{3}, \frac{19}{3}\right)$ from the line L along the line $\frac{3x-11}{2} = \frac{3y-11}{1} = \frac{3z-19}{2}$ is equal to

(1) 4 (2) 6 (3) 5 (4) 3

[JEE Main, 9th April 2024, Evening Shift]

FPR: 2

Class: XII

Chapter: Three Dimensional Geometry

Subtopic: Equation of Line

MATHEMATICS

Part-B

Q.21 to Q. 30 are Numerical Value Type Questions (+4, -1)

Attempt any 5 out of 10 Questions

21. Let the inverse trigonometric functions take principal values. The number of real solutions of the equation $2\sin^{-1}x+3\cos^{-1}x=\frac{2\pi}{5}$, is _____.

[JEE Main, 9th April 2024, Evening Shift]

FPR: 1

Class: XII

Chapter: Inverse Trigonometric Equations

Subtopic: Domain and Range of a ITF

22. The square of the distance of the image of the point (6, 1, 5) in the line $\frac{x-1}{3}=\frac{y}{2}=\frac{z-2}{4}$, from the origin is _____.

[JEE Main, 9th April 2024, Evening Shift]

FPR: 2

Class: XII

Chapter: Three Dimensional Geometry

Subtopic: Image, Foot of Perpendicular and Perpendicular Distance from Line

MATHEMATICS

23. Consider the matrices: $A=\begin{bmatrix}2 & -5\\ 3 & m\end{bmatrix}, B=\begin{bmatrix}20\\ m\end{bmatrix}$ and $X=\begin{bmatrix}x\\ y\end{bmatrix}$. Let the set of all m, for which the system of equation $AX=B$ has a negative solution (i.e., $x<0$ and $y<0$), be the interval (a,b). Then $8\int_a^b |A|\,dm$ is equal to

[JEE Main, 9th April 2024, Evening Shift]

FPR: 2

Class: XII

Chapter: Matrices

Subtopic: Product of Matrices

24. The number of integers, between 100 and 1000 having the sum of their digits equals to 14, is _____.

[JEE Main, 9th April 2024, Evening Shift]

FPR: 5

Class: XI

Chapter: Permutations and Combinations

Subtopic: Coefficient Method

MATHEMATICS

25. If $\left(\frac{1}{\alpha+1}+\frac{1}{\alpha+2}+\ldots\ldots+\frac{1}{\alpha+1012}\right)-\left(\frac{1}{2\cdot1}+\frac{1}{4\cdot3}+\frac{1}{6\cdot5}+\ldots\ldots+\frac{1}{2024\cdot2023}\right)=\frac{1}{2024}$, then α is equal to

[JEE Main, 9th April 2024, Evening Shift]

FPR: 4

Class: XI

Chapter: Sequence and Series

Subtopic: Miscellaneous/Mixed

26. Let the set of all values of p, for which $f(x) = (p^2 - 6p + 8)(\sin^2 2x - \cos^2 2x) + 2(2 - p)x + 7$ does not have any critical point, be the interval (a, b). Then 16 ab is equal to _____.

[JEE Main, 9th April 2024, Evening Shift]

FPR: 4

Class: XII

Chapter: Application of Derivatives

Subtopic: Critical and Stationary Points

MATHEMATICS

27. Consider the circle C: $x^2 + y^2 = 4$ and the parabola P: $y^2 = 8x$. If the set of all values of α, for which three chords of the circle C on three distinct lines passing through the point $(\alpha, 0)$ are bisected by the parabola P is the interval (p, q), then $(2q - p)^2$ is equal to _____.

[JEE Main, 9th April 2024, Evening Shift]

FPR: 3

Class: XI

Chapter: Parabola

Subtopic: Chords on Parabola

28. For a differentiable function $f: R \to R$, suppose $f'(x) = 3f(x) + \alpha$, where $\alpha \in R$, $f(0) = 1$ and $\lim\limits_{x \to -\infty} f(x) = 7$. Then $9f(-\log_e 3)$ is equal to _____.

[JEE Main, 9th April 2024, Evening Shift]

FPR: 5

Class: XII

Chapter: Differential Equation

Subtopic: Solution of Differential Equation: Variable Separable Form

MATHEMATICS

29. Let A = {(x, y) : 2x + 3y = 23, x, y ∈ N} and B = {x: (x, y) ∈ A}. Then the number of one-one functions from A to B is equal to _____.

[JEE Main, 9th April 2024, Evening Shift]

FPR: 2

Class: XII

Chapter: Relations and Functions

Subtopic: Classification of Functions(One-One, Many-One, Into and Onto)

30. Let A, B and C be three points on the parabola $y^2 = 6x$ and let the line segment AB meet the line L through C parallel to the x-axis at the point D. Let M and N respectively be the feet of the perpendiculars from A and B on L. Then $\left(\frac{AM \cdot BN}{CD}\right)^2$ is equal to _____.

[JEE Main, 9th April 2024, Evening Shift]

FPR: 2

Class: XI

Chapter: Parabola

Subtopic: Tangent and Normal on Parabola