MATHEMATICS

1. $\lim_{x \to \frac{\pi}{2}} \left(\frac{\int_{x^3}^{(\pi/2)^3} \left(\sin\left(2t^{1/3}\right) + \cos\left(t^{1/3}\right)\right) dt}{\left(x - \frac{\pi}{2}\right)^2} \right)$ is equal to

(1) $\frac{9\pi^2}{8}$ (2) $\frac{3\pi^2}{2}$ (3) $\frac{5\pi^2}{9}$ (4) $\frac{11\pi^2}{10}$

[JEE Main, 9th April 2024, Evening Shift]

FPR: 4

Class: XII

Chapter: Definite Integration

Subtopic: Leibnitz Theorem

FORMULA / CONCEPT

Leibnitz Theorem

If $F(x) = \int_{g(x)}^{h(x)} f(t)dt$, then $F'(x) = f(h(x))h'(x) - f(g(x)g'(x)$

Answer: (1)

Solution:

Using (L' Hospital Rule)

$$= \lim_{x \to \frac{\pi}{2}} \frac{-(\sin 2x + \cos x) \cdot 3x^2}{2\left(x - \frac{\pi}{2}\right)}$$

(Using Leibnitz Theorem)

Again, using L' Hospital rule,

$$= \lim_{x \to \frac{\pi}{2}} \frac{(-2\cos 2x + \sin x)3x^2 + 6x(-\sin 2x - \cos x)}{2}$$

$$= \frac{(-2\cos\pi + \sin\pi/2)3\frac{\pi^2}{4} + 6\times\frac{\pi}{2}\left(-\sin\pi - \cos\frac{\pi}{2}\right)}{2} = \frac{(2+1)3\pi^2 + 0}{8} = \frac{9\pi^2}{8}$$

MATHEMATICS

2. The sum of the coefficient of $x^{2/3}$ and $x^{-2/5}$ in the binomial expansion of $\left(x^{2/3}+\frac{1}{2}x^{-2/5}\right)^9$ is

(1) $63/16$ (2) $21/4$ (3) $69/16$ (4) $19/4$

[JEE Main, 9th April 2024, Evening Shift]

FPR: 3

Class: XI

Chapter: Binomial Theorem

Subtopic: General Term and Coefficients

FORMULA / CONCEPT

General Term

General term in the expansion of $(a + b)^n$ is $T_{r+1} = {}^nC_r\, a^{n-r}\, b^r$

Answer: (2)

Solution:

$$\left(x^{2/3}+\frac{1}{2}x^{-2/5}\right)^9$$

$$T_{r+1} = {}^9C_r\left(x^{2/3}\right)^{9-r}\cdot\left(\frac{1}{2}x^{-2/5}\right)^r$$

$$= {}^9C_r\left(\frac{1}{2}\right)^r\cdot x^{\frac{18-2r}{3}-\frac{2r}{5}}$$

For coeff. of $x^{2/3}$, $\frac{90-16r}{15}=\frac{2}{3}$

16 r = 80

r = 5

For coeff. of $x^{-2/5}$, $\frac{90-16r}{15}=\frac{-2}{5}$

16 r = 96

r = 6

$$\text{Sum of coff} = {}^9C_5\times\frac{1}{32}+{}^9C_6\,\frac{1}{64}$$

$$=\frac{63}{16}+\frac{21}{16}\Rightarrow\frac{84}{16}$$

$$=\frac{21}{4}$$

MATHEMATICS

3. The area (in square units) of the region enclosed by the ellipse $x^2 + 3y^2 = 18$ in the first quadrant below the line $y = x$ is

(1) $\sqrt{3}\pi + \frac{3}{4}$ (2) $\sqrt{3}\pi + 1$ (3) $\sqrt{3}\pi$ (4) $\sqrt{3}\pi - \frac{3}{4}$

[JEE Main, 9th April 2024, Evening Shift]

FPR: 4

Class: XI

Chapter: Ellipse

Subtopic: Auxiliary Circle and Parametric Equation

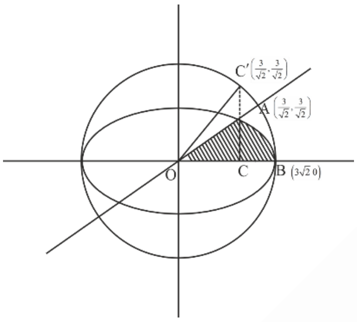
FORMULA / CONCEPT

Ratio of area inscribed in ellipse & area formed by corresponding points in its auxiliary circle

$$= \frac{\text{Semi – Minor Axis}}{\text{Semi – Major Axis}}$$

Answer: (3)

Solution:



$$E : x^2 + 3y^2 = 18 \,\&\, y = x$$

$$\frac{x^2}{(3\sqrt{2})^2} + \frac{y^2}{(\sqrt{6})^2} = 1$$

Solving above two equations

$$x = y = \frac{3}{\sqrt{2}}$$

$$A\left(\frac{3}{\sqrt{2}}, \frac{3}{\sqrt{2}}\right)$$

Now, equation of auxiliary circle of ellipse is $x^2 + y^2 = \left(3\sqrt{2}\right)^2 = 18$

$$\Rightarrow \quad C'\left(\frac{3}{\sqrt{2}}, \frac{3\sqrt{3}}{2}\right)$$

MATHEMATICS

$$\angle C'OC = \theta = \tan^{-1}\left(\frac{\frac{3\sqrt{3}}{\sqrt{2}}}{\frac{3}{\sqrt{2}}}\right)$$

$$\Rightarrow \theta = \tan^{-1}\left(\sqrt{3}\right) = \frac{\pi}{3}$$

Comparing equation of ellipse with $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$,

We get $a = 3\sqrt{2}$ & $b = \sqrt{6}$

Now, Required area

$$= \frac{b}{a} \times (\text{area of sector } OC'BO)$$

$$= \frac{b}{a} \times \frac{1}{2} r^2 \theta = \frac{\sqrt{6}}{3\sqrt{2}} \times \frac{1}{2} \times \left(3\sqrt{2}\right)^2 \times \frac{\pi}{3}$$

$$= \sqrt{3}\pi$$

MATHEMATICS

4. Let $\alpha, \beta; \alpha > \beta$, be the roots of the equation $x^2 - \sqrt{2}x - \sqrt{3} = 0$. Let $P_n = \alpha^n - \beta^n, n \in \mathbb{N}$. Then $\left(11\sqrt{3} - 10\sqrt{2}\right) P_{10} + \left(11\sqrt{2} + 10\right) P_{11} - 11P_{12}$ is equal to

(1) $10\sqrt{3}P_9$ (2) $11\sqrt{2}P_9$ (3) $10\sqrt{2}P_9$ (4) $11\sqrt{3}P_9$

[JEE Main, 9th April 2024, Evening Shift]

FPR: 2

Class: XI

Chapter: Quadratic Equation

Subtopic: Newton's Formula

FORMULA / CONCEPT

Newton's Formula

If α, β are the roots of a quadratic equation $ax^2 + bx + c = 0$ satisfying the relation $P_n = \lambda\alpha^n \pm \mu\beta^n$, then we have $a P_n + b P_{n-1} + c P_{n-2} = 0$

Answer: (1)

Solution:

$x^2 - \sqrt{2}x - \sqrt{3} = 0$

$P_n = \alpha^n - \beta^n$

$P_n - \sqrt{2}P_{n-1} - \sqrt{3}P_{n-2} = 0$

For n = 12

$P_{12} = \sqrt{3}P_{10} + \sqrt{2}P_{11}$ ….(1)

For n = 11

$P_{11} = \sqrt{3}P_9 + \sqrt{2}P_{10}$ …(2)

Now, Given expression,

$11\sqrt{3}P_{10} - 10\sqrt{2}P_{10} + 11\sqrt{2}P_{11} + 10P_{11} - 11P_{12}$

$= 11\sqrt{3}P_{10} - 10\sqrt{2}P_{10} + 11\sqrt{2}P_{11} + 10\sqrt{3}P_9 + 10\sqrt{2}P_{10} - 11\sqrt{3}P_{10} - 11\sqrt{2}P_{11}$ [using (1) & (2)]

$= 10\sqrt{3}P_9$

MATHEMATICS

5. Let $\vec{a}=2\hat{i}+\alpha\hat{j}+\hat{k}, \vec{b}=-\hat{i}+\hat{k}, \vec{c}=\beta\hat{j}-\hat{k}$, where α and β are integers and $\alpha\beta=-6$. Let the values of the ordered pair (α,β) for which the area of the parallelogram of diagonals $\vec{a}+\vec{b}$ and $\vec{b}+\vec{c}$ is $\frac{\sqrt{21}}{2}$, be (α_1,β_1) and (α_2, β_2). Then $\alpha_1^2+\beta_1^2-\alpha_2\beta_2$ is equal to:

(1) 19 (2) 17 (3) 24 (4) 21

[JEE Main, 9th April 2024, Evening Shift]

FPR: 4

Class: XII

Chapter: Vector Algebra

Subtopic: Cross Product

FORMULA / CONCEPT

Cross Product

If $\vec{a}=a_1\hat{i}+a_2\hat{j}+a_3\hat{k}$ and $\vec{b}=b_1\hat{i}+b_2\hat{j}+b_3\hat{k}$, then $\vec{a}\times\vec{b}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ a_1 & a_2 & a_3\\ b_1 & b_2 & b_3\end{vmatrix}$.

Answer: (1)

Solution:

Area of Parallelogram = $\frac{1}{2}\left|(\vec{a}+\vec{b})\times(\vec{b}+\vec{c})\right|=\frac{\sqrt{21}}{2}$

$$(\vec{a}+\vec{b})\times(\vec{b}+\vec{c})=\begin{vmatrix}i & j & k\\ 1 & \alpha & 2\\ -1 & \beta & 0\end{vmatrix}$$

$$=-2\beta\hat{i}-2\hat{j}+(\beta+\alpha)\hat{k}$$

$$\left|(\vec{a}+\vec{b})\times(\vec{b}+\vec{c})\right|=\sqrt{4\beta^2+4+\beta^2+\alpha^2+2\alpha\beta}=\sqrt{21}$$

$$5\beta^2+\alpha^2+4-12=21$$

$$\alpha^2+5\beta^2=29$$

Also,

$a\beta=-6$

$$\beta=\frac{-6}{\alpha}$$

$$\alpha^2+\frac{180}{\alpha^2}=29$$

MATHEMATICS

$\alpha^4 - 29\alpha^2 + 180 = 0$

$\alpha^4 - 20\alpha^2 - 9\alpha^2 + 180 = 0$

$(\alpha^2 - 20)(\alpha^2 - 9) = 0$

$\alpha = 3, -3$ as $\alpha \in I$

$\beta = -2, 2$

Now, $\alpha_1^2 + \beta_1^2 - \alpha_2\beta_2$

$= 9 + 4 + 6 = 19$

MATHEMATICS

6. Between the following two statements :

Statement I : Let $\vec{a} = \hat{i} + 2\hat{j} - 3\hat{k}$ and $\vec{b} = 2\hat{i} + \hat{j} - \hat{k}$. Then the vector $\vec{r}$ satisfying $\vec{a} \times \vec{r} = \vec{a} \times \vec{b}$ and $\vec{a} \cdot \vec{r} = 0$ is of magnitude $\sqrt{10}$.

Statement II : In a triangle $ABC, \cos 2A + \cos 2B + \cos 2C \geq -\frac{3}{2}$.

(1) Statement I is correct but Statement II is incorrect.

(2) Statement I is incorrect but Statement II is correct.

(3) Both Statement I and Statement II are correct.

(4) Both Statement I and Statement II are incorrect.

[JEE Main, 9th April 2024, Evening Shift]

FPR: 5

Class: XII

Chapter: Vector Algebra

Subtopic: Miscellaneous/Mixed

FORMULA / CONCEPT

Dot and Cross Product

Answer: (2)

Solution:

Given, $\vec{a} \times \vec{r} = \vec{a} \times \vec{b}$

$\vec{a} \times \left(\vec{r} - \vec{b}\right) = 0$

$\therefore \vec{r} - \vec{b} = \lambda \vec{a}$

$\vec{r} = \vec{b} + \lambda \vec{a}$

Now,

$\vec{a} \cdot \vec{r} = 0$

$\vec{a} \cdot \vec{b} + \lambda |\vec{a}|^2 = 0$

$7 + 14\lambda = 0$

$\lambda = -\frac{1}{2}$

$\vec{r} = 2\hat{i} + \hat{j} - \hat{k} - \frac{1}{2}\left(\hat{i} + 2\hat{j} - 3\hat{k}\right)$

$\vec{r} = \frac{3}{2}\hat{i} + \frac{1}{2}\hat{k}$

$$|\vec{r}| = \sqrt{\frac{9}{4}+\frac{1}{4}} = \frac{\sqrt{10}}{2}$$

Statement - I is Incorrect

Since, minimum value of cos2A + cos2B + cos2C holds for equilateral triangle.

Statement-II is correct.

MATHEMATICS

7. Let z be a complex number such that the real part of $\frac{z-2i}{z+2i}$ is zero. Then, the maximum value of $\left|z-(6+8i)\right|$ is equal to

(1) 10 (2) 12 (3) 8 (4) ∞

[JEE Main, 9th April 2024, Evening Shift]

FPR: 4

Class: XI

Chapter: Complex Numbers

Subtopic: Triangular Inequality

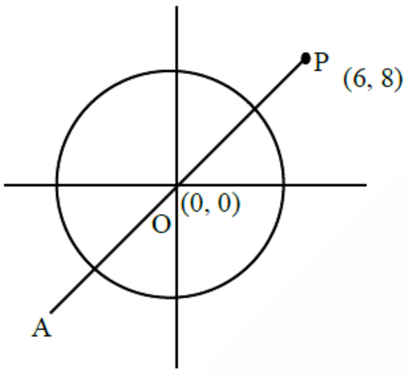
FORMULA / CONCEPT

1) $\text{Re}(z)=0 \Rightarrow z+\bar{z}=0$

2) $\left|z_1-z_2\right| \le \left|z_1\right|+\left|z_2\right|$

Answer: (2)

Solution:



$\Rightarrow \frac{z-2i}{z+2i}+\frac{\bar{z}+2i}{\bar{z}-2i}=0$

$\Rightarrow (z-2i)(\bar{z}-2i)+(\bar{z}+2i)(z+2i)=0$

$\Rightarrow 2(z\bar{z}-4)=0 \Rightarrow |z|=2$

Now, $\left|z-(6+8i)\right| \le |z|+|6+8i|$

$\le 2 + 10$

≤ 12

MATHEMATICS

8. $\lim_{x\to 0}\dfrac{e-(1+2x)^{\frac{1}{2x}}}{x}$ is equal to

(1) 0 (2) e (3) $e-e^2$ (4) $\dfrac{-2}{e}$

[JEE Main, 9th April 2024, Evening Shift]

FPR: 2

Class: XII

Chapter: Limits

Subtopic: Series Expansion

FORMULA / CONCEPT

Series Expansion

$$(1+x)^{1/x}=e\left(1-\frac{x}{2}+\frac{11x^2}{24}-...\right), \text{ for } |x|<1$$

Answer: (2)

Solution:

Let 2x = t

$$\lim_{t\to 0}\frac{e-(1+t)^{1/t}}{t/2}$$

$$=2\lim_{t\to 0}\frac{e-e\left(1-\frac{t}{2}+\frac{11}{24}t^2.....\right)}{t}$$

$$=2\lim_{t\to 0}\frac{e-e+\frac{et}{2}-\frac{11e}{24}t^2+......}{t}$$

$$=2\times\frac{e}{2}=e$$

MATHEMATICS

9. Let the foci of a hyperbola H coincide with the foci of the ellipse $E:\frac{(x-1)^2}{100}+\frac{(y-1)^2}{75}=1$ and the eccentricity of the hyperbola H be the reciprocal of the eccentricity of the ellipse E. If the length of the transverse axis of H is α and the length of its conjugate axis is β, then $3\alpha^2+2\beta^2$ is equal to

(1) 205 (2) 225 (3) 242 (4) 237

[JEE Main, 9th April 2024, Evening Shift]

FPR: 2

Class: XI

Chapter: Hyperbola

Subtopic: General Terms

FORMULA / CONCEPT

Co-ordinates of Foci of Ellipse and Hyperbola

Answer: (2)

Solution:

$$E:\frac{(x-1)^2}{100}+\frac{(y-1)^2}{75}=1$$

Given, $e_H=\frac{1}{e_E}$

$$e_E^2=1-\frac{3}{4}=\frac{1}{4}$$

$$e_E=\frac{1}{2}$$

$$\therefore e_H=2$$

Transverse axis $=\alpha\Rightarrow a=\frac{\alpha}{2}$

conjugate axis $=\beta\Rightarrow b=\frac{\beta}{2}$

$$e_H^2=1+\frac{\beta^2}{\alpha^2}$$

$$4\alpha^2=\alpha^2+\beta^2\Rightarrow 3\alpha^2=\beta^2 \quad \ldots\ldots(1)$$

$$ae=5$$

$$\frac{\alpha}{2}\times 2=5\Rightarrow \alpha=5$$

From equation (1), $\beta^2=75$

$$\therefore 3\alpha^2+2\beta^2=75+150=225$$

MATHEMATICS

10. The integral $\int_{1/4}^{3/4} \cos\left(2\cot^{-1}\sqrt{\frac{1-x}{1+x}}\right)dx$ is equal to

(1) 1/2 (2) –1/2 (3) –1/4 (4) ¼

[JEE Main, 9th April 2024, Evening Shift]

FPR: 4

Class: XII

Chapter: Definite Integration

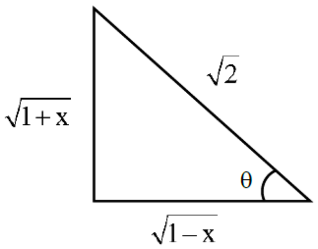
Subtopic: Direct Integration

FORMULA / CONCEPT

Integration Using Substitution

Answer: (3)

Solution:



$$\int_{1/4}^{3/4} \cos\left(2\cot^{-1}\sqrt{\frac{1-x}{1+x}}\right)\cdot dx$$

Let $\cot^{-1}\sqrt{\frac{1-x}{1+x}} = \theta$

$$\cot\theta = \frac{\sqrt{1-x}}{\sqrt{1+x}}$$

$\therefore \quad \cos\theta = \sqrt{\frac{1-x}{2}}$

Now, $\int_{1/4}^{3/4} \cos 2\theta \cdot dx$

$$= \int_{1/4}^{3/4} \left(2\cos^2\theta - 1\right)dx$$

$$= \int_{1/4}^{3/4} \left(2\frac{(1-x)}{2} - 1\right)dx$$

$$= \int_{1/4}^{3/4} \left[(1-x)-1\right]dx$$

$$= \int_{1/4}^{3/4} -x dx = \left[-\frac{x^2}{2}\right]_{1/4}^{3/4} = -\frac{1}{2}\left[\frac{9}{16}-\frac{1}{16}\right]$$

$$= -\frac{1}{2}\times\frac{8}{16} \Rightarrow -\frac{1}{4}$$

MATHEMATICS

11. If $\log_e y = 3\sin^{-1}x$, then $\left(1-x^2\right)y'' - xy'$ at $x = \frac{1}{2}$ is equal to

(1) $3e^{\pi/6}$ (2) $3e^{\pi/2}$ (3) $9e^{\pi/6}$ (4) $9e^{\pi/2}$

[JEE Main, 9th April 2024, Evening Shift]

FPR: 2

Class: XII

Chapter: Methods of Differentiation

Subtopic: Differentiation Based on Basic Rules(Product, Quotient and Chain)

FORMULA / CONCEPT

Product Rule of Differentiation

$(fg)'(x) = f(x)g'(x) + f'(x)\, g(x)$

Answer: (4)

Solution:

$$\log_e y = 3\sin^{-1}x$$

$$y = e^{3\sin^{-1}x} \qquad \ldots(1)$$

differentiating w.r.t x,

$$y' = e^{3\sin^{-1}x} \cdot \frac{3}{\sqrt{1-x^2}}$$

$$\sqrt{1-x^2} \cdot y' = 3e^{3\sin^{-1}x}$$

Again, differentiating w.r.t x

$$\sqrt{1-x^2} \cdot y'' - \frac{xy'}{\sqrt{1-x^2}} = \frac{9e^{3\sin^{-1}x}}{\sqrt{1-x^2}}$$

$$\left(1-x^2\right)y'' - xy' = 9y$$

(From equation (1))

Now, from equation (1), at $x = \frac{1}{2}$

$$y = e^{\pi/2}$$

$$\therefore \left(1-x^2\right)y'' - xy' = 9e^{\pi/2}$$

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MATHEMATICS

12. If the variance of the frequency distribution

x	c	2 c	3 c	4 c	5 c	6 c
f	2	1	1	1	1	1

is 160, then the value of $c \in N$ is

(1) 6 (2) 7 (3) 5 (4) 8

[JEE Main, 9th April 2024, Evening Shift]

FPR: 1

Class: XI

Chapter: Statistics

Subtopic: Mean and Variance

FORMULA / CONCEPT

$$\text{Mean} = \frac{\sum f_i x_i}{\sum f_i} \text{ and Variance} = \frac{\sum f_i (x_i - \text{mean})^2}{\sum f_i}$$

Answer: (2)

Solution:

x	c	2 c	3 c	4 c	5 c	6 c
f	2	1	1	1	1	1

$$\bar{x} = \frac{\sum f_i x_i}{\sum f_i} = \frac{22c}{7}$$

$$\text{Now, } \operatorname{var}(x) = \frac{\sum f_i (x_i - \bar{x})^2}{\sum f_i}$$

$$(x_i - \bar{x})^2 : \frac{225c^2}{49} \quad \frac{64c^2}{49} \quad \frac{c^2}{49} \quad \frac{36c^2}{49} \quad \frac{169c^2}{49} \quad \frac{400c^2}{49}$$

$$f \quad : \quad 2 \quad 1 \quad 1 \quad 1 \quad 1 \quad 1$$

$$\operatorname{var}(x) = \frac{c^2}{49}\left[\frac{450+64+1+36+169+400}{7}\right]$$

$$= \frac{1120c^2}{49 \times 7} = 160$$

$c^2 = 49$

$c = \pm 7$

$\therefore \quad c = 7$

$\because \quad c \in N$

MATHEMATICS

13. Let the range of the function $f(x)=\dfrac{1}{2+\sin 3x+\cos 3x}, x\in\mathbb{R}$ be $[a,b]$. If α and β are respectively the A.M. and the G.M. of a and b, then $\dfrac{\alpha}{\beta}$ is equal to

(1) $\sqrt{2}$ (2) π (3) $\sqrt{\pi}$ (4) 2

[JEE Main, 9th April 2024, Evening Shift]

FPR: 1

Class: XI

Chapter: Trigonometric Ratios and Identities

Subtopic: Trigonometric Functions

FORMULA / CONCEPT

Range of a sinθ + b cosθ is $\left[-\sqrt{a^2+b^2}, \sqrt{a^2+b^2}\right]$

Answer: (1)

Solution:

$$f(x)=\frac{1}{2+\sin 3x+\cos 3x}$$

We know that,

$$-\sqrt{2}\le \sin 3x+\cos 3x\le\sqrt{2}$$

$$2-\sqrt{2}\le 2+\sin 3x+\cos 3x\le 2+\sqrt{2}$$

$$\frac{1}{2+\sqrt{2}}\le\frac{1}{2+\sin 3x+\cos 3x}\le\frac{1}{2-\sqrt{2}}$$

$$\therefore a=\frac{1}{2+\sqrt{2}} \text{ or } \frac{2-\sqrt{2}}{2}$$

$$b=\frac{1}{2-\sqrt{2}} \text{ or } \frac{2+\sqrt{2}}{2}$$

$$\alpha= \text{ A. M of a \& b } =\frac{\frac{2-\sqrt{2}}{2}+\frac{2+\sqrt{2}}{2}}{2}=1$$

$$\beta= \text{ G. M of a \& b } =\sqrt{\frac{2-\sqrt{2}}{2}\times\frac{2+\sqrt{2}}{2}}=\frac{1}{\sqrt{2}}$$

$$\therefore \frac{\alpha}{\beta}=\frac{1}{\frac{1}{\sqrt{2}}}=\sqrt{2}$$

MATHEMATICS

14. Let $B=\begin{bmatrix}1 & 3\\ 1 & 5\end{bmatrix}$ and A be a 2 × 2 matrix such that $AB^{-1} = A^{-1}$. If $BCB^{-1} = A$ and $C^4 + \alpha C^2 + \beta I = O$, then $2\beta - \alpha$ is equal to

(1) 16 (2) 8 (3) 2 (4) 10

[JEE Main, 9th April 2024, Evening Shift]

FPR: 5

Class: XII

Chapter: Matrices

Subtopic: Characteristic Equation of a Square Matrix

FORMULA / CONCEPT

Characteristic equation of square matrix A is $|A - \lambda I| = 0$.

Answer: (4)

Solution:

$BCB^{-1} = A$

$BCB^{-1}\ BCB^{-1} = A\cdot A$

$BCICB^{-1} = A\cdot A$

$B^{-1}(BC^{-1}B^{-1})B = B^{-1}A\cdot AB$

$C^2 = B^{-1}AAB$ ...(1)

Now, $AB^{-1} = A^{-1}$

$AB^{-1}A = A^{-1}\cdot A = I$

$A^{-1}AB^{-1}A = A^{-1}\cdot I$

$B^{-1}A = A^{-1}$ ...(2)

From equation (1) & (2)

$C^2 = A^{-1}\cdot AB$

$C^2 = B$

Now, characteristic equation of C^2 is,

$|C^2 - \lambda I| = 0$

$|B - \lambda I| = 0$

$$\begin{vmatrix}1-\lambda & 3\\ 1 & 5-\lambda\end{vmatrix} = 0$$

$\lambda^2 - 6\lambda + 2 = 0$

$B^2 - 6B + 2 = 0$

$C^4 - 6C^2 + 2I = 0$

$\therefore \quad \alpha = -6, \beta = 2$

So, $2\beta - \alpha = 10$

MATHEMATICS

15. The value of the integral $\int_{-1}^{2} \log_e\left(x+\sqrt{x^2+1}\right)dx$ is

(1) $\sqrt{2}-\sqrt{5}+\log_e\left(\frac{7+4\sqrt{5}}{1+\sqrt{2}}\right)$

(2) $\sqrt{5}-\sqrt{2}+\log_e\left(\frac{7+4\sqrt{5}}{1+\sqrt{2}}\right)$

(3) $\sqrt{2}-\sqrt{5}+\log_e\left(\frac{9+4\sqrt{5}}{1+\sqrt{2}}\right)$

(4) $\sqrt{5}-\sqrt{2}+\log_e\left(\frac{9+4\sqrt{5}}{1+\sqrt{2}}\right)$

[JEE Main, 9th April 2024, Evening Shift]

FPR: 5

Class: XII

Chapter: Definite Integration

Subtopic: Definite Integration by Parts

FORMULA / CONCEPT

Integration By Parts

$$\int \mathrm{I}\cdot \mathrm{II}dx = \mathrm{I}\int \mathrm{II}dx - \int\left((\mathrm{I})'\int \mathrm{II}dx\right)dx$$

Answer: (3)

Solution:

$$\int_{-1}^{2} \log_e\left(x+\sqrt{x^2+1}\right)\cdot dx$$

$$\int_{-1}^{2} \log_e\left(x+\sqrt{x^2+1}\right)\cdot 1dx$$

$$\downarrow \qquad \qquad \downarrow$$
$$\mathrm{I} \qquad \qquad \mathrm{II}$$

Using by parts,

$$=\left[\log_e\left(x+\sqrt{x^2+1}\right)\cdot x\right]_{-1}^{2} - \int_{-1}^{2} \frac{1+\frac{2x}{2\sqrt{x^2+1}}}{x+\sqrt{x^2+1}}\cdot x\cdot dx$$

$$=2\log_e\left(2+\sqrt{5}\right)+\log_e\left(\sqrt{2}-1\right)-\int_{-1}^{2}\frac{\left(\sqrt{x^2+1}+x\right)x}{\left(\sqrt{x^2+1}\right)\cdot\left(x+\sqrt{x^2+1}\right)}dx$$

$$=\log_e\left[\frac{(2+\sqrt{5})^2}{\sqrt{2}+1}\right]-\int_{-1}^{2}\frac{x\cdot dx}{\sqrt{1+x^2}}$$

MATHEMATICS

$$=\log_e\left(\frac{9+4\sqrt{5}}{\sqrt{2}+1}\right)-\int_2^5\frac{dt}{2\sqrt{t}}\left(\begin{array}{c}\text{put } 1+x^2=t\\ x\cdot dx=\frac{dt}{2}\end{array}\right)$$

$$=\log_e\left(\frac{9+4\sqrt{5}}{\sqrt{2}+1}\right)-(\sqrt{t})_2^5$$

$$=\log_e\left(\frac{9+4\sqrt{5}}{\sqrt{2}+1}\right)-\sqrt{5}+\sqrt{2}$$

$$=\sqrt{2}-\sqrt{5}+\log_e\left(\frac{9+4\sqrt{5}}{\sqrt{2}+1}\right)$$

MATHEMATICS

16. If an unbiased dice is rolled thrice, then the probability of getting a greater number in the i^{th} roll than the number obtained in the $(i-1)^{th}$ roll, $i = 2,3$, is equal to

(1) 1/54 (2) 5/54 (3) 2/54 (4) 3/54

[JEE Main, 9th April 2024, Evening Shift]

FPR: 1

Class: XII

Chapter: Probability

Subtopic: Classical Definition

FORMULA / CONCEPT

Classical Definition of Probability

$$P(E) = \frac{n(E)}{n(S)}$$

Answer: (2)

Solution:

Let x_1, x_2, x_3 be the numbers on 1st, 2nd & 3rd throw respectively.

Given: $x_1 < x_2 < x_3$

$\therefore$ Favorable cases $= {}^6C_3$

Total cases $= 6^3 = 216$

$$\text{Prob} = \frac{{}^6C_3}{216} \Rightarrow \frac{20}{216} = \frac{5}{54}$$

MATHEMATICS

17. Let $a, ar, ar^2, \ldots\ldots$ be an infinite G.P. If $\sum_{n=0}^{\infty} ar^n = 57$ and $\sum_{n=0}^{\infty} a^3 r^{3n} = 9747$, then $a + 18r$ is equal to

(1) 46 (2) 38 (3) 27 (4) 31

[JEE Main, 9th April 2024, Evening Shift]

FPR: 4

Class: XI

Chapter: Sequence and Series

Subtopic: Geometric Progression

FORMULA / CONCEPT

Leibnitz Theorem

Sum of Infinite G.P. is $\frac{a}{1-r}$

Answer: (4)

Solution:

$$\sum_{n=0}^{\infty} ar^n = 57$$

$$\frac{a}{1-r} = 57 \qquad \ldots(1)$$

also,

$$\sum_{n=0}^{\infty} a^3 \cdot r^{3n} = 9747$$

$$\frac{a^3}{1-r^3} = 9747 \qquad \ldots(2)$$

Cubing equation (1)

$$\frac{a^3}{(1-r)^3} = (57)^3 \qquad \ldots(3)$$

Now, dividing eq (2) & (3)

$$\frac{\frac{a^3}{(1-r)(1+r+r^2)}}{\frac{a^3}{(1-r)^3}} = \frac{9747}{57 \times 57 \times 57}$$

$$\frac{1+r^2-2r}{1+r+r^2} = \frac{1}{19}$$

$19+19r^2-38r=1+r+r^2$

$18r^2-39r+18=0$

$6r^2-13r+6=0$

$6r^2-9r-4r+6=0$

$3r(2r-3)-2(2r-3)=0$

$(2r-3)(3r-2)=0$

$r=\frac{3}{2},\frac{2}{3}$

Taking $r=\frac{2}{3}$ [$\because$ r < 1, for ∞ sum of G.P.]

From equation (1) $a=57\times\frac{1}{3}\Rightarrow 19$

$\therefore a+18r=19+18\times\frac{2}{3}\Rightarrow 19+12=31$

MATHEMATICS

18. Two vertices of a triangle ABC are A(3, –1) and B(–2, 3), and its orthocentre is P(1, 1). If the coordinates of the point C are (α, β) and the centre of the circle circumscribing the triangle PAB is (h, k), then the value of (α + β) + 2(h + k) equals

(1) 5 (2) 51 (3) 15 (4) 81

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FPR: 3

Class: XI

Chapter: Straight Line

Subtopic: Centres of a Triangle

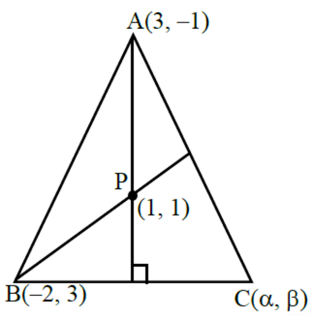
FORMULA / CONCEPT

"Orthocentre" of a triangle is intersection point of three altitudes.

"Circumcentre" of a triangle is intersection point of three perpendicular bisectors.

Answer: (1)

Solution:



Equation of

$$PC: \; y-1=\frac{5}{4}(x-1) \left(\because m_{AB}=-\frac{4}{5}\right)$$

$4y - 4 = 5x - 5$

$5x - 4y = 1$ …(1)

Equation of BC: $y - 3 = 1(x + 2)$

($\because$ $m_{AP} = -1$).

$x - y = -5$ …(2)

Solving equation (1) & (2), $x = 21$, $y = 26$

$\therefore$ $\alpha = 21, \beta = 26 \Rightarrow (\alpha + \beta) = 47$

Now, equation of ⊥ bisector of AP:

$y - 0 = 1(x - 2)$

$x - y = 2$ …(3)

Equation of ⊥ bisector of

$$AB: y - 1 = \frac{5}{4}\left(x - \frac{1}{2}\right) \quad \ldots(4)$$

Solving equation (3) & (4),

$$x = -\frac{19}{2}, y = -\frac{23}{2}$$

i.e $\quad 2h = -19,\ 2k = -23$

$\therefore \quad 2(h + k) = -42$

So, $\quad (\alpha + \beta) + 2(h + k) = 47 - 42 = 5$

MATHEMATICS

19. Let $\int_0^x \sqrt{1-(y'(t))^2}\,dt = \int_0^x y(t)\,dt, 0 \le x \le 3, y \ge 0, y(0) = 0$. Then at $x = 2, y'' + y + 1$ is equal to

(1) 1 (2) 2 (3) $\sqrt{2}$ (4) ½

[JEE Main, 9th April 2024, Evening Shift]

FPR: 2

Class: XII

Chapter: Definite Integration

Subtopic: Leibnitz Theorem

FORMULA / CONCEPT

Leibnitz Theorem

If $F(x) = \int_{g(x)}^{h(x)} f(t)\,dt$, then $F'(x) = f(h(x))h'(x) - f(g(x)g'(x)$

Answer: (1)

Solution:

$$\int_0^x \sqrt{1-(y'(t))^2}\,dt = \int_0^x y(t)\,dt$$

Differentiating both sides w.r.t x.

$$\sqrt{1-(y'(x))^2}\cdot 1 = y(x)\cdot 1$$

$$1-(y'(x))^2 = (y(x))^2$$

$$y'(x) = \sqrt{1-y^2(x)} \quad \Rightarrow (y'(x))^2 + (y(x))^2 = 1$$

Again Differentiating both sides, we get

$2(y'(x))\, y''(x) + 2(y(x))(y'(x)) = 0$

$\Rightarrow y''(x) + y(x) = 0$

$\Rightarrow y'' + y + 1 = 1$

MATHEMATICS

20. Consider the line L passing through the points $(1,2,3)$ and $(2,3,5)$. The distance of the point $\left(\frac{11}{3},\frac{11}{3},\frac{19}{3}\right)$ from the line L along the line $\frac{3x-11}{2}=\frac{3y-11}{1}=\frac{3z-19}{2}$ is equal to

(1) 4 (2) 6 (3) 5 (4) 3

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FPR: 2

Class: XII

Chapter: Three Dimensional Geometry

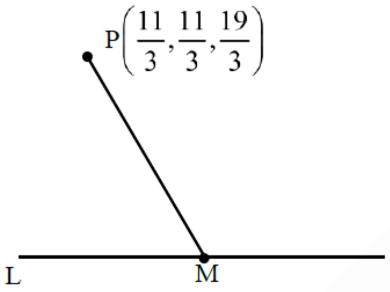
Subtopic: Equation of Line

FORMULA / CONCEPT

If direction ratios of two lines are proportional, then the lines are parallel.

Answer: (4)

Solution:



Equation in of tine L passing through (1, 2, 3) & (2, 3, 5)

$$\frac{x-1}{1}=\frac{y-2}{1}=\frac{z-3}{2}=\lambda$$

M$(\lambda + 1, \lambda + 2, 2\lambda + 3)$

DR's of PM: $\frac{3\lambda-8}{3},\frac{3\lambda-5}{3},\frac{6\lambda-10}{3}$

DR's of given line: $\frac{2}{3},\frac{1}{3},\frac{2}{3}$

$$\therefore \frac{3\lambda-8}{2}=\frac{3\lambda-5}{1}=\frac{6\lambda-10}{2}$$

$$3\lambda - 8 = 6\lambda - 10 \Rightarrow \lambda=\frac{2}{3}$$

$$B\left(\frac{5}{3},\frac{8}{3},\frac{13}{3}\right),\ AB=\sqrt{\frac{36+9+36}{9}}=3$$

MATHEMATICS

21. Let the inverse trigonometric functions take principal values. The number of real solutions of the equation $2\sin^{-1}x + 3\cos^{-1}x = \frac{2\pi}{5}$, is ______.

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FPR: 1

Class: XII

Chapter: Inverse Trigonometric Equations

Subtopic: Domain and Range of a ITF

FORMULA / CONCEPT

Range of $\cos^{-1}x$ is $[0, \pi]$

Answer: (0)

Solution:

$$2\sin^{-1}x + 2\cos^{-1}x + \cos^{-1}x = \frac{2\pi}{5}$$

$$2\left(\frac{\pi}{2}\right) + \cos^{-1}x = \frac{2\pi}{5}$$

$$\cos^{-1}x = \frac{2\pi}{5} - \pi$$

$$\cos^{-1}x = \frac{-3\pi}{5}$$

But $\cos^{-1} x \in [0, \pi]$

$\therefore$ No real value of x

No solution

MATHEMATICS

22. The square of the distance of the image of the point (6, 1, 5) in the line $\frac{x-1}{3}=\frac{y}{2}=\frac{z-2}{4}$, from the origin is ______.

[JEE Main, 9th April 2024, Evening Shift]

FPR: 2

Class: XII

Chapter: Three Dimensional Geometry

Subtopic: Image, Foot of Perpendicular and Perpendicular Distance from Line

FORMULA / CONCEPT

Image of a Point about a line.

Answer: (62)

Solution:

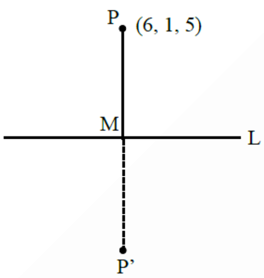

Image of point (6, 1, 5) in the line

$\frac{x-1}{3}=\frac{y}{2}=\frac{z-2}{4}=\lambda$

$x = 3\lambda + 1, y = 2\lambda, z = 4\lambda + 2$

$M(3\lambda + 1, 2\lambda, 4\lambda + 2)$

$\overrightarrow{PM}=(3\lambda-5)\hat{i}++(2\lambda-1)\hat{j}+(4\lambda-3)\hat{k}$

Now, $\overrightarrow{PM}\perp$ to line L

$\therefore \overrightarrow{PM}\cdot\left(3\hat{i}+2\hat{j}+4\hat{k}\right)=0$

$9\lambda - 15 + 4\lambda - 2 + 16\lambda - 12 = 0$

$29\lambda = 29 \Rightarrow \lambda = 1$

M(4, 2, 6)

P′ (2, 3, 7) → Image of P

Square of distance of P′ from origin is equal to

$\left(\sqrt{(2-0)^2+(3-0)^2+(7-0)^2}\right)^2 \Rightarrow 4 + 9 + 49 \Rightarrow 62$

MATHEMATICS

23. Consider the matrices : $A=\begin{bmatrix}2 & -5\\ 3 & m\end{bmatrix}, B=\begin{bmatrix}20\\ m\end{bmatrix}$ and $X=\begin{bmatrix}x\\ y\end{bmatrix}$. Let the set of all m, for which the system of equation $AX=B$ has a negative solution (i.e., $x<0$ and $y<0$), be the interval (a,b). Then $8\int_a^b |A|dm$ is equal to

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FPR: 2

Class: XII

Chapter: Matrices

Subtopic: Product of Matrices

FORMULA / CONCEPT

Solving matrix equation AX = B

Answer: (450)

Solution:

$AX = B$

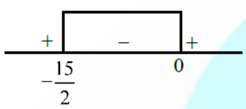
$X = A^{-1} B$

$$X=\frac{1}{2m+15}\begin{bmatrix}m & 5\\ -3 & 2\end{bmatrix}\begin{bmatrix}20\\ m\end{bmatrix}$$

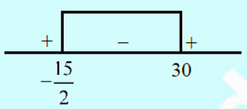
$$X=\frac{1}{2m+15}\begin{bmatrix}25m\\ 2m-60\end{bmatrix}$$

$$x=\frac{25m}{2m+15}, y=\frac{2m-60}{2m+15}$$

Now, $x<0$ and $y<0$



and



$M \in \left(-\frac{15}{2}, 0\right)$ $\quad$ $M \in \left(-\frac{15}{2}, 30\right)$

$$\therefore m \in \left(-\frac{15}{2}, 0\right)$$

$$a=-\frac{15}{2},\ b=0$$

Now, $8\int_a^b |A|dm$

$$= 8\int_{\frac{-15}{2}}^{0}(2m+15)\,dm$$

$$= 8\left[m^2+15m\right]_{-15/2}^{0}$$

$$= -8\left[\frac{225}{4}-\frac{225}{2}\right] = -8\times\left(-\frac{225}{4}\right) = 450$$

MATHEMATICS

24. The number of integers, between 100 and 1000 having the sum of their digits equals to 14, is ______.

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FPR: 5

Class: XI

Chapter: Permutations and Combinations

Subtopic: Coefficient Method

FORMULA / CONCEPT

Coefficient Method

Coefficient of n^r in $(1-x)^{-n} = {}^{n+r-1}C_r$

Answer: (70)

Solution:

$x_1 + x_2 + x_3 = 14$, where $1 \le x_1 \le 9$, $0 \le x_2, x_3 \le 9$

Coefficient of x^{14} in $\left(x^1 + x^2 + \ldots x^9\right)\left(x^0 + x^1 + x^2 + \ldots . + x^9\right)^2$

$$= x\left(1 + x + \ldots x^8\right)\left(1 + x + x^2 + \ldots x^9\right)^2$$

$$= x\left(\frac{1-x^9}{1-x}\right)\cdot\left(\frac{1-x^{10}}{1-x}\right)^2$$

$$= x\cdot\left(1-x^9\right)\left(1-x^{10}\right)^2\cdot(1-x)^{-3}$$

$$= x.\left(1-x^9\right)\left(1-2x^{10}+x^{20}\right)(1-x)^{-3}$$

$$= x.\left(1-2x^{10}+x^{20}-x^9+2x^{19}-x^{29}\right)(1-x)^{-3}$$

$$= x\left(1-2x^{10}-x^9+\ldots.\right)(1-x)^{-3} \quad \{\text{ignoring higher power}\}$$

No. of integer $= {}^{3+13-1}C_{13} - 2\left({}^{3+3-1}C_3\right) - \left({}^{3+4-1}C_4\right)$

$$= {}^{15}C_{13} - 2\left({}^{5}C_3\right) - {}^{6}C_4 \Rightarrow 105 - 20 - 15 \Rightarrow 70$$

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MATHEMATICS

25. If $\left(\frac{1}{\alpha+1}+\frac{1}{\alpha+2}+\ldots\ldots+\frac{1}{\alpha+1012}\right)-\left(\frac{1}{2\cdot 1}+\frac{1}{4\cdot 3}+\frac{1}{6\cdot 5}+\ldots\ldots+\frac{1}{2024\cdot 2023}\right)=\frac{1}{2024}$, then α is equal to

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FPR: 4

Class: XI

Chapter: Sequence and Series

Subtopic: Miscellaneous/Mixed

FORMULA / CONCEPT

Completing the series

Answer: (1011)

Solution:

$$\left(\frac{1}{\alpha+1}+\frac{1}{\alpha+2}+\ldots\ldots+\frac{1}{\alpha+1012}\right)-\left(\frac{1}{2\cdot 1}+\frac{1}{4\cdot 3}+\frac{1}{6\cdot 5}+\ldots\ldots+\frac{1}{2024\cdot 2023}\right)=\frac{1}{2024}$$

↓

Let it be z

Considering,

$$Z=\frac{1}{2.1}+\frac{1}{4.3}+\frac{1}{6.5}+\ldots\frac{1}{2024.2023}$$

$$Z=1-\frac{1}{2}+\frac{1}{3}-\frac{1}{4}+\frac{1}{5}-\frac{1}{6}+\ldots\frac{1}{2023}-\frac{1}{2024}$$

$$Z=\left(1+\frac{1}{3}+\frac{1}{5}+\ldots\ldots\frac{1}{2023}\right)-\left(\frac{1}{2}+\frac{1}{4}+\frac{1}{6}+\ldots\frac{1}{2024}\right)$$

$$Z=\left(1+\frac{1}{3}+\frac{1}{5}+\ldots\frac{1}{2023}\right)-\frac{1}{2}\left(1+\frac{1}{2}+\frac{1}{3}+\ldots\frac{1}{2012}\right)$$

Adding & subtracting $\left(\frac{1}{2}+\frac{1}{4}+\ldots\frac{1}{2022}\right)$

$$Z=\left(1+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\ldots\cdot\frac{1}{2023}\right)-\frac{1}{2}\left(1+\frac{1}{2}+\frac{1}{3}+\ldots\frac{1}{2012}\right)-\frac{1}{2}\left(1+\frac{1}{2}+\frac{1}{3}+\ldots\frac{1}{2011}\right)$$

$$Z=\left(1+\frac{1}{2}+\frac{1}{3}+\ldots\frac{1}{2023}\right)-\left(1+\frac{1}{2}+\frac{1}{3}+\ldots\frac{1}{2011}\right)-\frac{1}{2024}$$

$$Z=\frac{1}{1012}+\frac{1}{1013}\cdots\frac{1}{2023}-\frac{1}{2024}$$

Now, put z' in equation (1)

MATHEMATICS

$$\left(\frac{1}{\alpha+1}+\frac{1}{\alpha+2}+\ldots\frac{1}{\alpha+10/2}\right)-\left(\frac{1}{1012}+\frac{1}{1013}+\ldots\frac{1}{2023}\right)+\frac{1}{2024}$$

Now, comparing above equation with equation (1)

$\alpha = 1011$

MATHEMATICS

26. Let the set of all values of p, for which $f(x) = (p^2 - 6p + 8)(\sin^2 2x - \cos^2 2x) + 2(2 - p)x + 7$ does not have any critical point, be the interval (a, b). Then 16 ab is equal to ______.

[JEE Main, 9th April 2024, Evening Shift]

FPR: 4

Class: XII

Chapter: Application of Derivatives

Subtopic: Critical and Stationary Points

FORMULA / CONCEPT

Critical Points

Values of ‘x’ where either f(x) = 0 or $f'(x)$ fails to exist are critical points.

Answer: (252)

Solution:

$$f(x) = (p^2 - 6p + 8)(\sin^2 2x - \cos^2 2x) + 2(2 - p)x + 7$$

$$f(x) = (p^2 - 6p + 8)(-\cos 4x) + 2(2 - p)x + 7$$

$$f'(x) = 4\sin 4x(p^2 - 6p + 8) + 2(2 - p) \neq 0$$

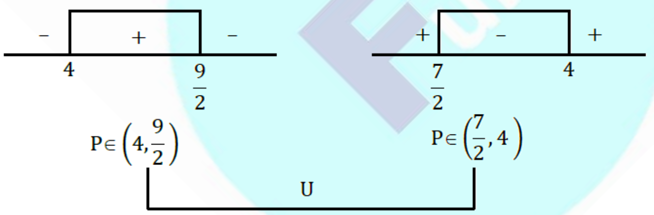
$$\sin 4x \neq \frac{2(p-2)}{4(p-2)(p-4)}$$

$$\frac{(p-2)}{2(p-2)(p-4)} > 1 \text{ or } \frac{p-2}{2(p-2)(p-4)} < -1 \; (p \neq 2)$$

$$\frac{1}{2p-8} - 1 > 0 \text{ or } \frac{1}{2p-8} + 1 < 0$$

$$\frac{1-2p+8}{2(p-4)} > 0 \text{ or } \frac{1+2p-8}{2(p-4)} < 0$$

$$\frac{a-2p}{2(p-4)} > 0 \text{ or } \frac{2p-7}{2(p-4)} < 0$$



$$P \in \left(\frac{7}{2}, \frac{9}{2}\right)$$

$a=\frac{7}{2},\ b=\frac{9}{2}$

$\therefore 16ab=16\times\frac{7}{2}\times\frac{9}{2}=252$

MATHEMATICS

27. Consider the circle C: $x^2 + y^2 = 4$ and the parabola P: $y^2 = 8x$. If the set of all values of α, for which three chords of the circle C on three distinct lines passing through the point (α, 0) are bisected by the parabola P is the interval (p, q), then $(2q - p)^2$ is equal to _______.

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FPR: 3

Class: XI

Chapter: Parabola

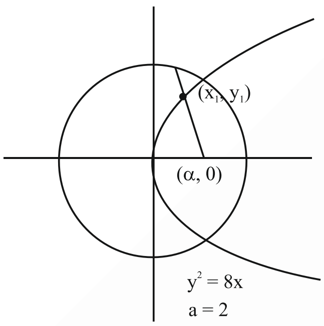
Subtopic: Chords on Parabola

FORMULA / CONCEPT

Equation of chord whose mid-point is given $T = S_1$

Answer: (80)

Solution:



$T = S_1$

$xx_1 + yy_1 = x_1^2 + y_1^2$

It passes through $(\alpha, 0)$

$\therefore \alpha x_1 = x_1^2 + y_1^2$

$\alpha(2t^2) = 4t^4 + 16t^2 \; (x_1 = 2t^2, y_1 = 4t)$

$\alpha = 2t^2 + 8$

$t^2 = \dfrac{\alpha - 8}{2} \quad \because \; t^2 > 0$

$\Rightarrow \quad \alpha > 8$

Also,

$4t^4 + 16t^2 - 4 < 0$ (point (x_1, y_1) lies inside the circle)

$t^4 + 4t - 1 < 0$

$(t^2 + 2 + \sqrt{5})(t^2 + 2 - \sqrt{5}) < 0$

$\Rightarrow \; t^2 + 2 - \sqrt{5} < 0$

$\Rightarrow \; t^2 < -2 + \sqrt{5}$

MATHEMATICS

$\Rightarrow \quad \alpha = 2t^2 + 8 < 4 + 2\sqrt{5}$

$\therefore \quad \alpha \in \left(8, 4 + 2\sqrt{5}\right)$

$\therefore \alpha \in \left(8, 4 + 2\sqrt{5}\right)$

$\therefore (2q - p)^2 = 80$

MATHEMATICS

28. For a differentiable function f : R→R, suppose f′ (x) = 3f(x) + α, where α ∈ R, f(0) = 1 and $\lim_{x \to -\infty} f(x) = 7$.

Then $9f(-\log_e 3)$ is equal to ______.

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FPR: 5

Class: XII

Chapter: Differential Equation

Subtopic: Solution of Differential Equation: Variable Separable Form

FORMULA / CONCEPT

Solution of differential equation : Variable separable forms

Answer: (61)

Solution:

$f'(x) = 3f(x) + \alpha$

$$\frac{dy}{dx} = 3y + \alpha$$

$$\frac{dy}{3y + \alpha} = dx$$

Integrating both the sides

$$\int \frac{dy}{3y + \alpha} = \int dx$$

$$\frac{1}{3}\ln|3y + \alpha| = x + c$$

$\ln|3y + \alpha| = 3x + 3c$

$$|3y + \alpha| = e^{3x+3c}$$

When $x \to -\infty$ $y = 7$

$3y + \alpha = 0$

$\alpha = -21$

$$\frac{1}{3}\ln|3y - 21| = x + c$$

Now, when x = 0, y = 1

$$\frac{1}{3}\ln 18 = c$$

$$\frac{1}{3}\ln|3y - 21| = x + \frac{1}{3}\ln 18$$

$$\frac{1}{3}\ln\left(\frac{7-y}{6}\right)=x$$

Now, f(–ln 3)

$$\left[\ln\left(\frac{7-y}{6}\right)=-3\ln 3\right]$$

$$\left[\frac{7-y}{6}=\frac{1}{27}\right]$$

$$7-y=\frac{2}{9}$$

$$y=7-\frac{2}{9}=\frac{61}{9}$$

$$\therefore \quad 9f(-\ln 3)=9\times\frac{61}{9}=61$$

MATHEMATICS

29. Let A = {(x, y) : 2x + 3y = 23, x, y ∈ N} and B = {x: (x, y) ∈ A}. Then the number of one-one functions from A to B is equal to ______.

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FPR: 2

Class: XII

Chapter: Relations and Functions

Subtopic: Classification of Functions(One-One, Many-One, Into and Onto)

FORMULA / CONCEPT

Classification of Functions

If n(A) = n(B) = m, then number of one-one functions from A→B is m!

Answer: (24)

Solution:

2x + 3y = 23,

x, y ∈ N

A = {(1, 7), (4, 5), (7, 3), (10, 1)}

B = {1, 4, 7, 10}

then no. of one-one functions = 4! = 24

MATHEMATICS

30. Let A, B and C be three points on the parabola $y^2 = 6x$ and let the line segment AB meet the line L through C parallel to the x-axis at the point D. Let M and N respectively be the feet of the perpendiculars from A and B on L. Then $\left(\frac{AM \cdot BN}{CD}\right)^2$ is equal to ______.

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FPR: 2

Class: XI

Chapter: Parabola

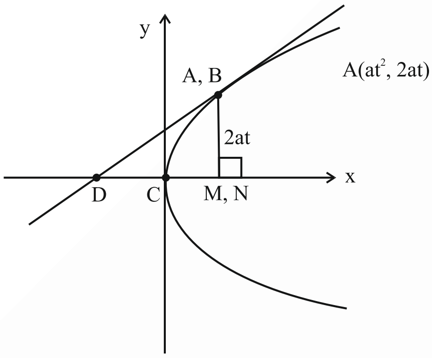
Subtopic: Tangent and Normal on Parabola

FORMULA / CONCEPT

Equation of tangent at a point A(t) on parabola $y^2 = 4ax$ is $yt = x + at^2$.

Answer: (36)

Solution:



Considering A, B same point on parabola & M, N same foot of perpendiculars on x-axis.

Let C be at origin (0, 0)

Equation of tangent at A(t) is

$yt = x + at^2$

$D(-at^2, 0)$

$$\left(\frac{AM \cdot BM}{CD}\right)^2 = \left(\frac{2at \cdot 2at}{at^2}\right)^2$$

$= (4a)^2$

Comparing $y^2 = 4ax$ & $y^2 = 6x$, we get

$\boxed{4a = 6}$

Now, $\left(\frac{AM \cdot BN}{CD}\right)^2 = 6^2 = 36$